

International Mathematical Union

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When did you realize that you want to be mathematician? Who and what inspired you to become a mathematician? Was it an easy or difficult decision for you?

I've always taken things one step at a time and just chosen the most appealing next option at each career step. So it was probably only around the end of my PhD when I decided for myself that I really wanted to be a mathematician - before then it always felt possible I might prefer doing something else, even if I was always heading in the direction of mathematics. I probably wasn't really aware that being a mathematician was even a career option until quite late on. However, to an outside observer I think I would have looked very keen on mathematics from a young age, so I suspect everyone else was aware that I would probably end up as a mathematician a long time before I was.

Because I took everything in small steps, there was never a single big or particularly difficult decision. I was very fortunate to be in a system where I didn't face any significant barriers and several people had done similar things before, and so I was free to just choose the most appealing option to me at each stage.

What kind of mathematics did you like the best in school and at the university? Do you have a your favorite math problem from back then? How did you find "your" area of mathematics? What makes it particularly attractive for you?

Number theory has always had a special appeal to me - it wasn't taught at school, but I learnt various bits and pieces on my own through independent reading. I remember when I applied to university we were asked to put down a couple of our favourite mathematical subjects to be asked questions on, and I put down number theory which I think was pretty unusual. Fortunately I wasn't asked too many questions on it, since my self-taught knowledge was not very comprehensive! My university courses confirmed to me that number theory was what I was really interested in.

It is always a bit difficult to reverse-engineer the reasons why I find something interesting, but lots of the questions in number theory have a particular beauty to me. There are many simple to state problems in number theory which seem completely fundamental yet their answers require a huge amount of sophisticated ideas from all over mathematics. It is this combination of simplicity, importance and depth which really appeals.

How do you choose the problems to work on?

Number theory is very fortunate to have a large number of high-profile open problems. When I'm looking for new problems I normally start by trying to understand some of the partial results towards one of these big famous problems, and then seeing if I can make some small

improvement on the technical aspect of these ideas. I find this approach allows me to find problems which are well-motivated (they are in some way linked to understanding a famous problem) but at the same time feasible (I am only really trying to understand pre-existing ideas, rather than attack a famous problem head-on).

When I'm more familiar with a sub-area, I often try to look for toy problems which explore the limitations of our current techniques. Many techniques are limited in that they cannot see a particular important feature of a problem, and so I like to try to come up with the simplest possible toy problem which has this feature. In the simple setting of the toy problem you can try to develop a new technique which is sensitive to the feature, with the hope being that there should be a more complicated version of the technique that can then say something new about the original problem.

Do you feel that the progress in mathematics is sometimes very fast and sometimes very slow? What do you when it goes fast? What do you when it is slow?

Yes! Both my own research and the field as a whole seems to have bursts of progress and activity, and then quieter periods.

It is always very exciting when there is a burst of activity. Often this is caused by one or two new ideas which overcome a limitation in our techniques, and these can apply in different forms to give progress on a number of problems. When this happens I try to keep up with all the developments as best as possible, and enjoy the ride!

Quieter periods are harder for me - it is important to stay motivated even when you've been stuck on the main problems you've been working on for months. I like to have a mix of projects at any one time, so if I'm stuck on my main projects I can work on something else where I have a better idea of how things should work out, or write up some ideas that I've had before but not got round to. This way I get to feel like I have made some concrete progress, even if it is not on my main projects, which helps keep me positive. It is in the nature of ambitious research that many projects will get stuck, but I also find that making sure I am genuinely interested in the problem in the first place also helps a lot to keep working at it.

What was the first real mathematical result you obtained? Tell us about the result itself, the circumstances, and what effect it had on you. Can you tell us about your biggest "Eureka!" moment?

The first result I proved in my PhD was a bound on the mean-square gap between prime numbers. Strangely I didn't feel particularly different about it compared with solving difficult questions from courses, even though this was a research problem rather than something with a known solution. I think it did take a lot of pressure off the rest of my PhD, since I felt comfortable that I had already done some research and so could take more risks in other projects. It later turned out that this result had actually already been proven in the literature without me knowing it, but fortunately by the time I found out I was fully distracted with my next project and so I didn't mind too much.

The first result I proved which gained quite a bit of attention from the field was a new result on bounded gaps between primes just after my PhD. This was very exciting to me, because it came soon after the breakthrough work of Yitang Zhang and so there was already a lot of interest in this problem. However, along with the excitement came a lot of nerves - my result seemed almost too good to be true so I was very afraid of missing a mistake in my work, and it turned

out that Terence Tao had also independently come up with the same idea so I had some initial fears I was 'scooped'. Fortunately neither of these fears were realised!

What did you feel and what did you do when you learned about being awarded the prize?

I was in a bit of a daze for several days! I remember when the IMU president Carlos Kenig told me I had won, he asked me to confirm that I accepted the prize. I had a momentary panic that in my excitement I would mess up my words and accidentally decline the prize by mistake, so I said my confirmation to him bizarrely slowly to make sure I didn't mess it up.

I was obviously delighted to hear the news, but it took quite a while for it all to sink in, and so the week or so afterwards is a bit of a blur to me. Some aspects were certainly a bit surreal, but in a nice way. I had to keep the news secret from colleagues, so I had to act as if nothing had changed, which meant it sometimes felt as if I'd imagined it.

Who are the people who contributed the most to this success?

During my academic career, my PhD supervisor Roger Heath-Brown, my postdoc supervisor Andrew Granville and my colleague at Oxford Ben Green have all played very important roles in my development by mentoring me as I progress. I'm truly grateful to them - I owe them all a huge amount.

More widely, I've been supported by a huge number of people within the field. More senior established people have welcomed me into the field and made me feel at home. I've made a lot of friends, and have had a number of collaborations with amazing mathematicians. It is always extremely useful to see other people's intuition and to learn from them in collaborations.

What are the new horizons, new problems, new goals for you now?

I'm really looking forward to getting back into doing in-person mathematics after a long period of time online due to the pandemic. I think my research took a bit of a hit due to the disruption and lack of interaction, but I already have found that going for lunch and coffee with colleagues has helped envigorate me. I particularly look forward to meeting friends and collaborators at conferences in the future.

Do you draw inspiration from teaching mathematics? Do you wish people knew more about what is happening in mathematics? Is there something mathematicians should do to help people appreciate the importance and beauty of what they do?

I always find it surprising how teaching mathematics seems to be unexpectedly useful for research. The process of teaching forces me to reassess things that I thought I new pretty well, and several times this need to reassess has turned out to be a very useful exercise to get a new perspective or to see what is really important. This then helps both with being a clearer teacher and when similar ideas come up in research.

It is unfortunate that mathematics is a beautiful subject with wonderful ideas, but much of this is very hard to appreciate without formal training. I'd love it if there was a way to share with the wider public some of this beauty, and show that mathematics is about ideas not calculations. It is certainly something I strive to get across when teaching. I'm very grateful to various skilled popularisers of mathematics who do a great job of trying to bridge this gap to the general public!

The expansion of the internet has allowed for new ways of providing content to mathematically interested people in the general public. I think this is very good for mathematics in general, particularly when real mathematicians get involved.

How important are interactions with a computer for your work, now and in the future? Do you get more from interactions with a computer or interactions with people?

I use computers quite a lot to help my work, but only in a simple manner. Often to prove a result it is important to guess intermediate statements that would lead to the result (like stepping stones for crossing a river). Unfortunately I often guess intermediate results which turn out to be false, even though the main result is true! Rather than waste a day trying to prove some intermediate result which turns out to be false, it is often easy to check many cases on a computer which would normally tell me very quickly if my guess was false.

I imagine in the future computers might well be able to help rather more at this process of very quickly testing simple conjectures and doing well-understood basic mathematics. This would enable researchers to focus on thinking about higher-level ideas, which is often where the real ideas are required.

I certainly get rather more out of interacting with people than computers. Computers can help a lot at speeding up low-level technical ideas, but rarely help me much on higher-level understanding. When I talk with other mathematicians, it is often the complete opposite - we talk at a higher more abstract level, and can concentrate on the key ideas. This means that other mathematicians help me with the high-level thinking, which is where the key breakthroughs happen.

Outside of mathematics, what are your favorite things to do, interests, pursuits? Do you approach them as a mathematician, or are you happy to forget about mathematics while you are on a break?

One of the nice perks of being a mathematician is that you have the opportunity to travel to lots of interesting places as part of conferences and collaborations. I really like using this as an opportunity to take some time off from mathematics and explore other cities and cultures. Two things I particularly like doing when travelling is exploring coffee shops and taking photographs - I've got very into both coffee and photography over the past few years. In many ways these allow me to focus on something completely separate to mathematics and are a welcome break, but I still approach them in a similar way to mathematics - becoming a bit obsessive and totally focused on them!

As a prize winner, you also become an ambassador of mathematics to the society and its leaders. What would say in a meeting with, for instance, a Science and Education Minister?

There are two surprising facts which I view as vital for how to view mathematics in a modern society.

Firstly, that mathematics is the most powerful language mankind has for really understanding the world - be it economics, climate change, engineering or computer science. Therefore mathematics underlies the understanding of almost all aspects of our daily lives, and will only

grow in importance as the world and technology becomes more complicated, even if it isn't immediately visible.

Secondly, that applied or applicable mathematics cannot be separated from pure and abstract mathematics. It is amazing to me how many aspects of mathematics which were originally studied purely out of intellectual curiosity have later become vital to the modern world. Similarly many developments in pure mathematics have been inspired by ideas coming from the real world. It is clear that if abstract mathematics had been curtailed in favour of directly applicable mathematics, then we would have fewer rather than more applications of mathematics to the real world, and so these two aspects of mathematics need to grow harmoniously.